Indian J. Phys. 76A (4), 351-354 (2002)

IJP A
— an international journal

Temperature dependence of dc conductance of generalized Fibonacci lattice

C L Roy
Department of Physics & Meteorology, Indian Institute of Technology
Kharagpur 721 302, India

E-mail

Received 24 September 2001, accepted 9 April 2002

Abstract We have studied the dc conductance of a generalized Fibonacci superlattice (GFSL) at finite temperature, for a wide range of parameters relevant to the GFSL The circumstances which influence the dc conductance significantly are indicated Among other things, we find that GFSL generally behaves as a narrow gap semiconductor.

Keywords Fibonacci lattice, dc conductance, temperature dependence

PACS Nos. 72.10.-d, 73.21.Ac

1. Introduction

During nearly last two decades, studies of electronic properties of one-dimensional (1D) quasiperiodic (QP) systems like Fibonacci lattice [1-11] and Thus, Morse lattice [12-21] have been drawing widespread attention. The studies on Fibonacci lattice (FL) and Thue-Morse lattice (TML) stimulated interest respectively in Generalized Fibonacci (GFL) [22-25] and Generalized Thue-Morse lattice (GTML) [26-29]. Historically, the study of FL began earlier than TML and there have appeared, by now, several theoretical treatments [1-8, 10, 11] of FL which has also been realized experimentally [9] in the form of GaAs-AlAs superlattice.

The most exhaustively studied aspect in connection with QP system, is the so-called Landauer resistance (LR) [30] or equivalently, the dc conductance which is the reciprocal of LR. So far, the study of this aspect has remained largely confined to QP systems at absolute zero [7, 8, 11, 21, 23, 29], whereas the QP systems one treats in practice, are likely to be at finite temperatures. It is only during very recent years that the study of conductance of QP systems at finite temperatures started drawing attention [31, 32). The systems treated in [31] and [32] were FL. In view of the fact we indicated earlier regarding how studies of FL generated interest in GFL, the investigation of dc conductance of GFL seems worthwhile and the purpose of this communication is to report an effort in this direction.

* Corresponding Author

The organization of materials presented subsequently is as follows. In Section 2, we describe the model we have studied. In Section 3, we put forward the method we have followed for treating dc conductance of GFL at different temperatures. In Section 4, we project issues related to our numerical analysis. Finally, in Section 5, we incorporate a critical discussion about our results : among other things, this discussion includes (i) a comparison between temperature-dependence of dc conductance of FL and GFL and (ii) the feasibility of testing our findings about dc conductance of GFL experimentally.

2. The model and some related issues

We consider a generalized Fibonacci Superlattice (GFSL) consisting of two types of building blocks A and B, which are arranged according to the sequence $S_l^{m,n}$ shown below :

$$S_l^{m,n} = (S_{l-1})^m \, (S_{l-2})^n ; \;\; m, n = 1, 2, \ldots\ldots : l = 2, 3 \ldots \quad (1)$$

$$S_0 = A, \; S_1 = B .$$

The integer l is the order of the GFSL and the integers m and n are its degree. The number $F_l^{m,n}$ of elements in $S_l^{m,n}$, is given by

$$F_l^{m,n} = (F_{l-1})^m + (F_{l-2})^n , \quad (2)$$

where $F_0 = F_1 = 1$.

For the GFSL we have treated, A and B correspond to two kinds of rectangular potential barriers. The system constructed this way makes it similar to the most widely studied super lattice, namely, $GaAS - Ga_{1-x}Al_xAs$.. The Schrödinger equation describing the motion of the electron in our GFSL appears as

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar} W(x)\psi(x) = E\psi(x), \tag{3}$$

where

$$W(x) = \sum_{i=1}^{n} V(x - x_i),$$

E = Energy eigenvalue of the electron,

$$V(x - x_i) = V_i, \qquad for\ |x - x_i| \le b_i/2,$$

$$= 0 \qquad for\ |x - x_i| > b_i/2.$$

The potential barriers $V(x - x_i)$ are supposed to be located at $x = x_i$. The heights (V_i) and widths (b_i) of these barriers may correspond to A – type or B – type building blocks. For a A – type block at $x = x_i$, we assume that $V_i = V_A$ and $b_i = b_A$. Similarly, if a barrier corresponds to a B – type bloch, $V_i = V_B$ and $b_i = b_B$.

It may be noted that the Hamiltonian H corresponding to eq. (3) is a continuous Hamiltonian. This kind of Hamiltonian seems to be capable of taking account of realistic features better [18, 33] than the so called discretized Hamiltonian which depends on the use of tight binding approximation.

3. dc conductance of GFSL

Assuming that the potentials are non-overlapping, as is the case for semiconductor superlattice, eq. (3) can be exactly solved piece-wise using the well known formalism of transfer matrix, and the transmission of the electron across the GFSL can be obtained in terms of the elements of the transfer matrix. With the help of the transmission coefficient, we can evaluate the dimensionless dc conductance $\sigma(\mu, T)$ of our GFSL at a temperature T on the basis of the following widely used formula due to Engquist and Anderson [34]:

$$\sigma(\mu, T) = \frac{\int_0^{\infty} \left(-\frac{\partial f}{\partial E}\right) \tau(E) dE}{\int_0^{\infty} \left(-\frac{\partial f}{\partial E}\right) [1 - \tau(E)] dE} \tag{4}$$

where $f = \dfrac{1}{1 + \exp\left[(E - \mu)/kT\right]}$,

μ = Chemical potential,

$\tau(E)$ = Transmission coefficient of an electron with energy E, across the GFSL.

$\tau(E)$ generally appears as [1]

$$\tau(E) = \frac{1}{\left|M_l^{m,n}(1,2)\right|} \tag{5}$$

where $M_l^{m,n}$ is the (2×2) transfer matrix for the GFSL with order parameter l and degree parameters m and n. As is well known, the LR of a system of barriers is defined [30, 35] as the ratio of the reflection to the transmission coefficient of an electron travelling across the barriers. Consequently, the dc conductance $\sigma(T, \mu)$, which is the reciprocal of LR, is the ratio of the transmission to reflection coefficient of an electron passing across the barriers of our GFSL. In evaluating $\sigma(\mu, T)$, we inducted $\tau(E)$ *via* the element $M_l^{m,n}(1,2)$ of the transfer matrix $M_l^{m,n}$ which is obtained by following the procedure reported earlier [11, 24].

4. Numerical analysis

The main purpose of our numerical analysis is to evaluate over a wide range of temperatures for various values of parameters relevant to our GFSL. We briefly mention below the principal characteristics of our numerical analysis.

For studying the variation of $\sigma(\mu, T)$ with T for various values of μ, we have considered two cases of GFSL each with a fixed set of values of V_A, b_A, V_B and b_B, the results for $\sigma(\mu, T)$ corresponding to these situations are shown in Figures 1 and 2.

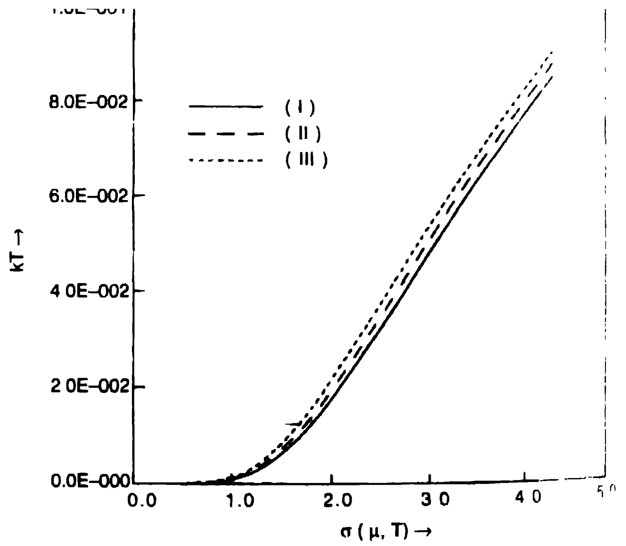

Figure 1. Graphs of $\sigma(\mu, T)$ *vs* kT for GFSL corresponding to the sequence $S_4^{1,2}$. For all the graphs, $b_A = 0.25$A, $b_B = 0.375$ A, $V_A = 80$ eV $V_B = 160$ eV, The values of μ for graphs I, II and III are respectively 3eV, 3.5 eV and 4 eV.

The transport of an electron through a system like the present GFSL; essentially corresponds to its tunnelling through rectangular barrier type potentials. An issue which is of great importance in regard to this kind of electronic transport, is related to investigating how it is affected when the width and height of the barriers are changed in a way as to keep its area constant; an

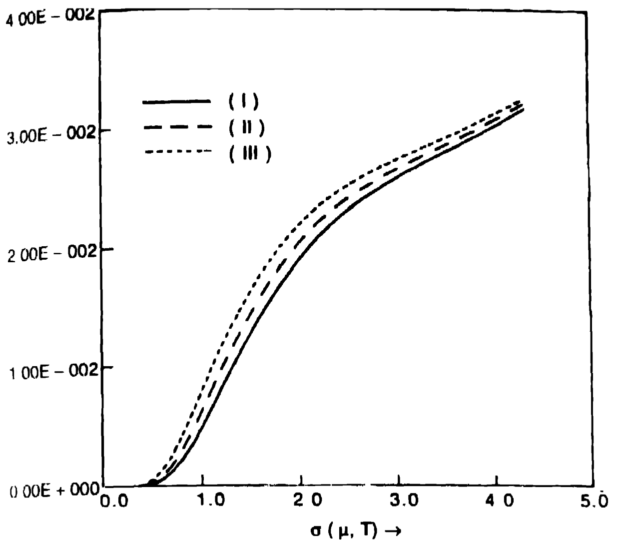

Figure 2. Graphs of $\sigma(\mu,T)$ *vs* kT for GFSL corresponding to the sequence $S_4^{1,3}$. For all the graphs, b_A = 1A, b_B = 1.5 A, V_A = 20eV, V_B = 40 eV. The value of μ are the same as those in Figure 1

investigation of this sort, indicates among other things, how the motion of the electron and related properties would appear when the potential barriers approach δ - function limits. To study this issue for our case, we have evaluated $\sigma(\mu,T)$ for one type of GFSL, choosing heights and widths of the barriers in the way just mentioned and considering two values of μ. The results relevant to $\sigma(\mu,T)$ thus evaluated are shown in Figures 3 and 4.

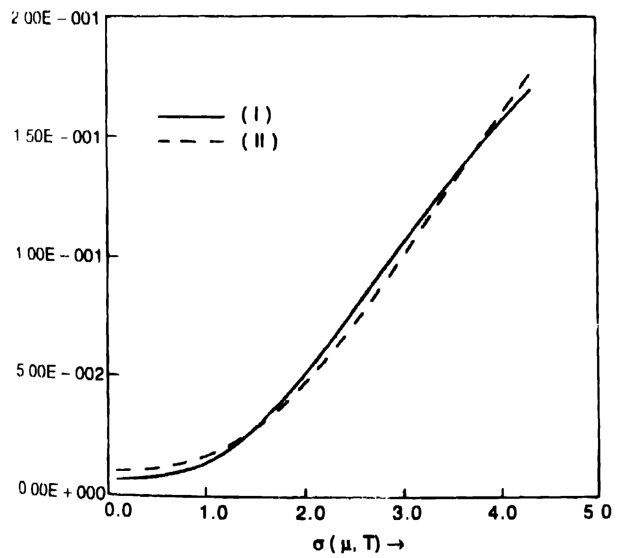

Figure 3. Graphs of $\sigma(\mu,T)$ *vs* kT for GFSL corresponding to the sequence $S_4^{1,2}$. For all the graph I, b_A = 0.5A, b_B = 0.75 A, V_A = 40eV, V_B = 80eV. For graph II, b_A = 0.25A, b_B = 0.375 A, V_A = 80eV, V_B = 160eV The value of $b_i V_i$ (i = A or B) is the same for both graphs. The value of μ for both the graphs is 3.0 × 10^{-12} erg.

The scale along kT axis for all the Figures is such that kT = 1 correspond to a temperature of 110° K. The total range of temperatures. We have considered is roughly 0° K to 550° K.

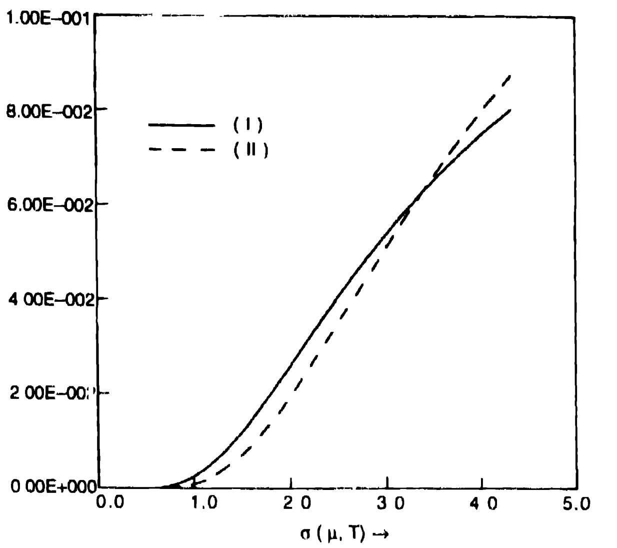

Figure 4. Same as graphs as in Figure 3, with μ = 3.5 × 10^{-12} erg The values of b_i and V_i (i = A or B) are the same as those in Figure 3.

5. Results and discussion

The graphs in Figures 1-4 lead to the following features.

(I) Looking at Figures 1-4, we observe that $\sigma(\mu,T)$ increase with temperature under all circumstances, remaining greater than zero above nearly 100° K. This behaviour indicates that the GFL we have studied, behaves as a narrow gap semiconductor ; the band gap of the system is likely to be around 0.01 ev which corresponds to 3/2kT with T = 100 K.

(II) The graphs in Figures 1 and 2 indicate that with increase of μ $\sigma(\mu,T)$ generally increases.

(III) Comparing the results in Figures 1 and 2, we note that other parameters remaining same, $\sigma(\mu,T)$ is larger for GFL with l = 4, m = 1, n = 2, than the one with l = 4, m = 1, n = 3. This situation suggests that $\sigma(\mu,T)$ decrease when the difference between order parameters increases ; this fact again indicates that the transmission coefficient τ (E), decreases when the difference between m and n increases.

(IV) The graphs in Figures 3 and 4 indicate that when V_i and b_i (i = A or B) are changed keeping ($V_i b_i$) a constant, there is virtually no effect on $\sigma(\mu,T)$ when μ = 3.5 × 10^{-12} ergs. However, when μ increases to 3.5 × 10^{-12} ergs, $\sigma(\mu,T)$ seems to depend on the choice of V_i and b_i ($V_i b_i$ = constant) ; specifically for this value of μ $\sigma(\mu,T)$ appears to be conspicuously more, over a quite wide range of temperatures, for higher values of b_A and b_B, *i.e.* when the potential barriers tend to become less like δ-functions.

(V) It is well known [10, 11, 24] by now that the energy spectrum of all kinds of QP systems consists of fragmented, allowed and forbidden regions and that τ (E) is negligible for E lying in the forbidden regions, while it is large for E in allowed regions. In view of these situations, the significant contributions to $\sigma(T,\mu)$ would come for values of E lying in the allowed regions of the energy spectrum of the GFL.

To end, we like to discuss following two issues.

(A) Comparison between $\sigma(T,\mu)$ of FL and GFL :

The dc conductance of FL at finite temperatures was studied earlier by some authors [31, 32]. The main qualitative features of our results for GFL are (i) the GFL behaves as a narrow gap semiconductor and (ii) $\sigma(T,\mu)$ generally increases with increase of μ. It appears that these two qualitative features are also present in $\sigma(T,\mu)$ for FL. However, from quantitative point of view, the features (i) and (ii) for FL differ from those of GFL. In respect of (i), the temperature at which $\sigma(T,\mu)$ starts becoming zero is more for FL than that for GFL. In respect of (ii), the increase of $\sigma(T,\mu)$ is more for FL than that for GFL. Primarily, the quantitative differences in respect of (i) and (ii) owe their origin to the difference between $\tau(E)$ of FL and GFL.

(B) Comparison, between our results and experimental findings :

As far as we know, no experimental study of $\sigma(T,\mu)$ of GFL (or FL) has yet been carried out. Hence, at the moment, we are not in a position to compare our results for $\sigma(T,\mu)$ of GFL with corresponding experimental findings. However, we feel that experimental study of $\sigma(T,\mu)$ of GFL is quite possible. For this purpose, one can fabricate GFL by using the method followed by authors of [9] for realizing FL in practice ; we feel that theoretical results for $\sigma(T,\mu)$ of GFL, like those of ours, would be helpful in interpreting the experimental findings which may appear in future in regard to $\sigma(T,\mu)$ of GFL.

Acknowledgments

The author is grateful to Professor H N Acharya, Head of the Department of Physics and Meteorology, Indian Institute of Technology, Kharagpur, and Prof. R N P Chowdhury, Department of Physics and Meteorology, Indian Institute of Technology, Kharagpur, for allowing me to use some facilities.

References

[1] S Ostlund, R Pandit, D Rand, H Schellnhuber and E D Siggia *Phys Rev. Lett.* **50** 1873 (1983)

[2] M Kohomoto, L P Kadanoff and C Tang *Phys Rev Lett* **50** 1870 (1983)

[3] M Kohomoto, B Sutherland and C Tang *Phys. Rev* **B35** 1020 (1987)

[4] Q Nill and F Nori *Phys. Rev. Lett.* **57** 2057 (1986)

[5] J A Ashraff and R B Stinchcombe *Phys. Rev.* **B37** 5723 (1988)

[6] Y Avishai and D Bernad *Phys. Rev.* **B41** 5492 (1990)

[7] J Kollar and A Suto *Phys Lett.* **A117** 203 (1986)

[8] S Das Sarma and X C Xie *Phys. Rev.* **B37** 1097 (1988)

[9] R Merlin, K Bajema, R Clarke, F Y Juang and P K Bhattacharya *Phys. Rev. Lett.* **1768** (1985)

[10] C L Roy and Arif Khan *Phys. Lett* **A196** 346 (1996)

[11] C L Roy and Arif Khan *Phys Rev.* **B49** 14979 (1994)

[12] F Axel, J P Allouche, M Kleman, M Medas-Frannce and J Peyriere *Physique Coll* **47** C3 181 (1986)

[13] D Levine and P Teinhardt *Phys Rev Lett* **57** 1157 (1986)

[14] R Riklund, M Severin and Y Liu *Int J Mod Phys* **B1** 121 (1987)

[15] M G Qin, H R Ma and C H Tsai *J Phys Condens Matter* 2 1059 (1990)

[16] C S Ryu, G Y Oh and M H Lee *Phys Rev* **B46** 4162 (1992)

[17] D Huang, G Gumbs and M Kolar *Phys Rev* **B46** 11479 (1992)

[18] C L Roy and Arif Khan *Solid State Commun.* **92** 241 (1994)

[19] R Merlin, K Bajema, J Nagle and K Ploog *J Physique Coll* 48 C5 5036 (1987)

[20] F Axel and H Terauchi *Phys Rev Lett* **66** 2223 (1991)

[21] C S Ryu, G Y G Y Oh and M H Lee *Phys. Rev* **B46** 5162 (1992)

[22] G Gumbs and M K Ali *Phys Rev Lett* **60** 1081 (1988)

[23] M Dulea, M Servin and R Riklund *Phys Rev* **B42** 3680 (1990)

[24] G Y Oh, C S Ryu and M H Lee *J Phys Condens Matter* 4 818 (1992)

[25] P K Ghosh *Phys. Lett* **A161** 153 (1991)

[26] C L Roy Chandan Basu and Arif Khan *Phys Lett* **A198** 42 (1995)

[27] M Kolar, M K Ali and F Nori *Phys Rev* **B43** 1034 (1991)

[28] Y Avishai and D Berend *Phys Rev* **B45** 2717 (1992)

[29] C L Roy, Arif Khan and Chandan Basu *J Phys. Condens Matter* 7 1843 (1995)

[30] R Landauer *Phil Mag* **21** 863 (1970)

[31] E Macia and F Dominguez-Adame *Phys Rev* **B49** 11578 (1994)

[32] C L Roy, Chandan Basu, F Dominguez-Adame and E Macia *Phys Stat Sol.* (b) **189** 193 (1995)

[33] M Baake, D Joseph and P Kramer *Phys Lett* **A168** 199 (1992)

[34] H L Engquist and P W Anderson *Phys Rev.* **B24** 1151 (1981)